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Reg. No. : .....

Name : .....

Third Semester B.Sc. Degree Examination, January 2023

First Degree Programme Under CBCSS

Mathematics

Complementary Course for Physics

MM 1331.1 : MATHEMATICS III – CALCULUS AND LINEAR ALGEBRA

(2018 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – A

All the **ten** questions are compulsory. They carry **1** mark each.

1. Define order of an ODE.
2. Check whether the equation $(3x + y)dx + xdy = 0$ is exact or not.
3. Write the general form of Euler's linear equation.
4. Prove that $div\vec{r} = 3$
5. State Green's theorem.
6. Find the average value of the function $f(x) = 1 - e^{-x}$ on (0, 1)
7. What are the fourier coefficients of an odd function $f(x)$ in the intererval (–l, l).
8. Find the matrix product of (0, 1) $\begin{bmatrix} -1 & 2 \\ 4 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ -2 \end{bmatrix}$

P.T.O.

9. Define Hermition matrix.

10. Define trace of a matrix.

(10 × 1 = 10 Marks)

SECTION – B

Answer **any eight** questions. These questions carry 2 marks each.

11. Solve $y'-xy^3=0$

12. Solve $x\frac{dy}{dx}+3x+y=0$

13. Find a particular integral of the equation $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=e^x$

14. Solve $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+y=0$

15. Calculate $\int_C \vec{F}.d\vec{r}$ where $\vec{F}=y^2\vec{i}-x^2\vec{j}$ along the parabola $y=4x^2$ from (0,0) to (1,4).

16. Check whether $\vec{F}=e^x\sin x\,\vec{i}+e^x\cos\vec{j}-2z\vec{i}\cos x\vec{k}$ is solenoidal.

17. Write the complex form of fourier series.

18. Write fourier cosine Transform.

19. Find the row reduced matrix of $\begin{pmatrix}2 & 0 & -1 & 2\\ 6 & 5 & 3 & 7\\ 2 & -1 & 0 & 4\end{pmatrix}$

20. Show that without computation $\begin{vmatrix}0 & 2 & -2\\ -2 & 0 & 4\\ 2 & -4 & 0\end{vmatrix}=0$

21. Find the angle between the lines $2x+6y-3z=0$ and $5x+2y-z=12$

22. Show that the functions $1,\ x,\ \sin x$ are linearly independent.

(8 × 2 = 16 Marks)

SECTION – C

Answer **any six** questions. These questions carry **4** marks each.

23. Solve $\frac{dy}{dx} = \frac{y}{x} + \tan(y/x)$

24. Solve $\frac{dy}{dx} = \frac{2x - 5y + 3}{2x + 4y - 6}$

25. Solve $xp^2 + 2xp - y = 0$

26. Evaluate the line integral $I = \oint x dy$, where C is the circle in the xy- plane C defined by $x^2 + y^2 + 2^2 = a^2$, $z = 0$.

27. A Vector field $\vec{a} = f(r)\vec{r}$ is spherically origin. Show that $\vec{a}$ is solenoidal if $f(r)$ is of the form Ar^{-3}

28. Given $f(x) = \begin{cases} 0 & 0 < x < l \\ 1 & l < x < 2l \end{cases}$

    Expand $f(x)$ is an exponential fourier series of period $2l$.

29. Prove that fourier expansion of $f(x) = \begin{cases} 0 & -\pi < x < 0 \\ 1 & 0 < x < \pi \end{cases}$ is

$$f(x) = \frac{1}{2} + \frac{2}{\lambda}\left(\frac{\sin x}{1} + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \cdots\right)$$

30. Find the rank of the matrix $\begin{pmatrix} 1 & 1 & 4 & 3 \\ 3 & 1 & 10 & 7 \\ 4 & 2 & 14 & 10 \\ 2 & 0 & 6 & 4 \end{pmatrix}$

31. Find the inverse of the matrix $\begin{pmatrix} -1 & 2 & 3 \\ 2 & 0 & -4 \\ -1 & -1 & 1 \end{pmatrix}$

(6 × 4 = 24 Marks)

SECTION – D

Answer **any two** questions. These question carries **15** marks each.

32 (a) Solve $\frac{d^2y}{dx^2} + 4y = x^2 \sin 2x$

(b) Solve $x^3\frac{d^2y}{dx^2} - (x^2 + xy)\frac{dy}{dx} + (y^2 + xy) = 0$

33 The vector field $\vec{F}$ given be $\vec{F} = (3x^2y^2 + y^3z + xe^x)\vec{i} + (3xy^2z + x^3z + ye^y)\vec{j} + (x^3y + y^3x + xy^2z^2)\vec{k}$. Calculate

(a) Directly, and

(b) By using stoke's theorem the value of the line integral $\int_L \vec{F}.d\vec{r}$, where L is the (three dimensioned) closed contour OABCDEO defined by sucessive vertices (0, 0, 0), (1, 0, 0), (1, 0, 1), (1, 1, 1), (1, 1, 0), (0, 1, 0), (0, 0, 0)

34 Let $f(x) = x^2, -\pi < x < \pi$ Find (a) a fourier sine series (b) a fourier cosine series (c) a complex exponential fourier series.

35 (a) Find the eigen values and eigen vectors of the matrix $\begin{pmatrix} 2 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & -1 \end{pmatrix}$

(b) Show that the product of two unitary matrices is unitary.

(2 × 15 = 30 Marks)
